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Derivative-based event-triggered control for networked systems with quantization

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ABSTRACT

This paper is devoted to the problem of derivative-based event-triggered control for linear networked systems with quantization. A novel type of event-triggered mechanism (ETM) is presented via involving a derivative term related with the Lyapunov functional. Inspired by this introduced derivative term, we call the proposed ETM as derivative-based event-triggered mechanism (DETM). In the conventional ETM, the triggering condition is always needed to be less than zero to ensure the system stability, while it is not required under the proposed DETM. Then, with the help of linear matrix inequality (LMI) technique, sufficient conditions for co-designing the triggering matrix and an event-triggered controller are derived. The corresponding results are also applied to event-triggered control systems with time-varying transmission delay. The validity of the developed approach is illustrated by some simulation examples.

1. Introduction

In networked systems, signals are usually exchanged among different system nodes via shared communication network. Many existing results on networked systems are based on periodic or time-triggered execution, which may cause the over-occupation of limited resources, such as channel bandwidth and node energy. There are some effective ways of saving communication resources, such as data quantization [1–3] and event-triggered communication scheme [4–6]. Over the past decade, considerable effort has been spent on event-triggered control strategy for its advantage of reducing data transmissions [7–11]. Under the ETM, new signal will be triggered and transmitted to update the controller only when it satisfies the prescribed triggering rule. Owing to this advantage, it has been applied widespread in robotics [12,13], wireless networks [14–16], smart grids [17,18] and so on. Regarding the design of ETM, some meaningful results have been reported, see [19–22] and references therein. Specifically, [19] studies the input-to-state stability of nonlinear systems via a continuous event-triggered mechanism. To exclude Zeno behavior induced by this mechanism, an exact lower bound of two adjacent triggering instants is also calculated. Based on this result, a novel event-triggered scheme by adding a dynamic variable is investigated in [20], where the triggering condition is not required to be always non-negative. Without requiring the Lyapunov function to be monotonically decreasing, [21] proposes an integral-based event-triggered control approach to ensure the system stability. To utilize the merit of continuous measurement and avoid Zeno behavior, a switching method, between periodic sampling and continuous ETM is addressed in [22].

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Without assuming that there exists a stabilization controller in the above results, [23] proposes a co-design approach to obtain the controller gain and triggering matrix. Based on this method, a great deal of outcomes about event-triggered control [24–28], filtering [29–31] and fault detection [32] have been reported. To mention a few, [24] studies the event-triggered H_∞ control issue of networked control systems with time-varying delay and packet dropout, where a maximum allowable number of consecutive packet dropout is derived. Different from some existing ETMs with a constant threshold, [25] designs an adaptive ETM that the triggering threshold can be regulated adaptively for nonlinear cascade system. In [29], an event-triggered filter is designed for linear sampled data systems. On the other hand, quantization is also able to reduce communication burden and investigated in [3,33,34]. For networked systems with quantization and Markov packet dropouts, [3] develops a new quantization structure to deal with the H_∞ quantized output control issue. In [33], a quantized H_∞ stabilization issue is addressed for singular networked systems with ETM and quantization. Considering the effect of stochastic cyber-attacks, [34] investigates the quantized stabilization issue of networked T-S fuzzy systems with a hybrid ETM, which is combined by time-triggered mechanism and ETM. Note that the triggering conditions in [33,34] are usually required to be negative except for the triggering instants. Actually, it is not necessary to make triggering condition always be negative based on the idea in [20]. To save more previous network resources, a possible solution is to integrate the state quantization and event-triggered scheme with the idea in [20]. However, this integration is nontrivial due to the following two aspects. First, a novel event-triggered scheme that the triggering condition is not always negative should be reconstructed. Second, a co-design method of the novel event-triggered scheme and the quantized controller needs further investigation.

Motivated by the aforementioned observations, the problem of derivative-based event-triggered control for linear networked systems with quantization is studied in this paper. First, a DETM including the conventional triggering condition and the derivative term related with the Lyapunov functional is proposed, which results in a less conservative triggering condition and a larger inter-event time. Second, adopting the constructive separation strategy in terms of Finsler lemma, the coupling between the controller gain and the Lyapunov variable is eliminated. Then, sufficient conditions formed by LMIs are obtained to co-design the controller gain and the triggering matrix. Moreover, the extension of the results for the networked systems considering time-varying transmission delay is also provided. Finally, simulations are executed to show the effectiveness of the derived results.

Notation: $\mathbb{R}^n$ is the Euclidean space with n dimension and $\mathbb{R}^{n\times m}$ is the set of $n \times m$ real matrices. $\mathcal{X}^T$ stands for the transpose of a matrix or vector $\mathcal{X}$. He$(\mathcal{X})$ equals to $\mathcal{X}^T + \mathcal{X}$. If a symmetric matrix $\mathcal{X}$ is positive (negative) definite, it is represented as $\mathcal{X} > 0$ (< 0).

2. Preliminaries

Consider a linear networked system described as

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are the state, the control input, respectively. A and B are known system parameters.

To decrease the 'unnecessary' signal transmission, ETM is utilized to select which signal is 'necessary' to be sent to the controller node. First, a normal ETM is given as

$$t_{k+1} = t_k + \sup\{t \in \mathbb{R} | g(t) < 0\}, \tag{2}$$

where

$$g(t) = e(t)^T \Phi e(t) - \sigma x^T(t) \Phi x(t),$$

$\sigma \in (0, 1)$, $e(t) = x(t) - x(t_k)$, $\Phi > 0$ represents the weighting matrix. Under this condition, a new event is generated once $g(t) \geq 0$.

For continuous-time ETM based on the relative error, the existence of a minimal inter-event time to remove Zeno behavior is an vital issue, which has been proved in [19]. Thus, we can derive the same result on the triggering condition (2) by a similar way.

In this paper, to further reduce the data transmissions, a DETM by introducing a derivative term is proposed as

$$t_{k+1} = t_k + \sup\{t \in \mathbb{R} | f(t) < 0\}, \tag{3}$$

where

$$f(t) = \theta f_1(t) + f_2(t),$$
$$f_1(t) = 2x^T(t) P \dot{x}(t),$$
$$f_2(t) = e^T(t) \Phi e(t) - \sigma x^T(t) \Phi x(t),$$

θ is a non-negative scalar, $\sigma \in (0, 1)$, $e(t) = x(t) - x(t_k)$, $\Phi > 0$, $P > 0$ means the Lyapunov variable matrix. Under the DETM, the derivative term $2x^T(t)P\dot{x}(t)$ is combined with the normal ETM (2) and a new event is triggered whenever $f(t) \geq 0$. The framework of the system with the proposed DETM and quantization is drawn in Fig. 1.

Remark 1. Compared with the normal ETM only using the current system state $x(t)$ and the error $e(t)$, a new derivative term related to the Lyapunov functional $V(t) = x^T(t)Px(t)$ is added into the proposed DETM. The main intuition of the

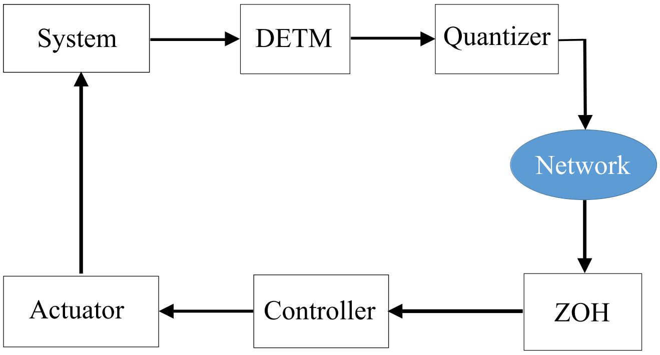

Fig. 1. The framework of event-triggered control system with quantization.

proposed DETM is that for ensuring the event-triggered control system to be stable, it does not require that $g(t)$ in (2) is always negative. With the help of the derivative term, the proposed triggering rule (3) is relaxed, which can reduce more data transmissions than the normal triggering rule (2).

Remark 2. The presented DETM (3) covers some existing ETMs as special cases. As $\theta = 0$, one has $f(t) = f_2(t) = g(t)$. Then (3) can be reduced to the ETM studied in [26] without considering Markovian jumps and the ETS in [30] with output matrix $C = I$.

Remark 3. If we choose $\theta \to \infty$, a limit case of (3) is expressed as

$$f_1(t) = \dot{V}(t) < 0, \ t \in \Omega, \tag{4}$$

which is the ETM:

$$-\delta x^T(t)Qx(t) + 2x^T(t)PBKe(t) < 0, \ \delta \in (0, \ 1) \tag{5}$$

proposed in [19] with $-Q = P(A+BK) + (A+BK)^T P$ and $\delta \to 1$. In this triggering rule, the inter-event time is increasing along with the increase of δ. When $\delta \to 1$, the largest inter-event time will be obtained by (5), which also equals to (4). This fact indicates that the proposed DETM (3) can produce larger inter-event time than that in [19].

Before the triggered signal is transmitted over the network channel, a logarithmic quantizer $q(x) = \begin{bmatrix} q_1(x_1) & \cdots & q_i(x_i) & \cdots & q_n(x_n) \end{bmatrix}$ is adopted, which is given as

$$q_i(x_i) = \begin{cases} \mathbb{Q}_l^{(i)}, & x_i > 0, \ \dfrac{\mathbb{Q}_l^{(i)}}{1+z_i} < x_i < \dfrac{\mathbb{Q}_l^{(i)}}{1-z_i} \\ 0, & x_i(t) = 0 \\ -q_i(-x_i), & x_i < 0 \end{cases} \tag{6}$$

where $z_i = \frac{1-\rho_i}{1+\rho_i}$, $\rho_i \in (0,1)$ is the quantizer density of $q_i(x_i)$. The set of quantization levels is presented by:

$$\begin{aligned} \mathcal{Q} &= \left\{ \pm\mathbb{Q}_l^{(i)}, \ \mathbb{Q}_l^{(i)} = \rho_i^l \mathbb{Q}_0^{(i)}, \ l = \pm 1, \pm 2, \cdots \right\} \\ &\cup \left\{ \pm\mathbb{Q}_0^{(i)} \right\} \cup \{0\}, \ 0 < \rho < 1, \ \mathbb{Q}_0^{(i)} > 0. \end{aligned} \tag{7}$$

Based on the above definition, the measurements with quantization effects $q(x(t))$ are formulated as

$$q(x(t)) = (I + \Delta_q)x(t) \tag{8}$$

where $\Delta_q = diag\{\Delta_{q_1}, \ldots, \Delta_{q_i}, \ldots, \Delta_{q_n}\}$, $\Delta_{q_i} \in [-z_i, \ z_i]$, $i = 1, 2, \ldots, n$. In this article, it is supposed that $z_i = z$ for analysis simplicity.

According to the above analysis, and considering the function of zero-order hold (ZOH), the control input of (1) among two adjacent events is expressed as

$$u(t) = Kq(x(t_k)) = K(I + \Delta_q)x(t_k) \tag{9}$$

for $t \in [t_k, \ t_{k+1})$, where K represents the controller gain to be solved.

Thus, the closed-loop system with the quantized controller (9) and the DETM (3) is established as:

$$\dot{x}(t) = Ax(t) + BK(I + \Delta_q)x(t) - BK(I + \Delta_q)e(t). \tag{10}$$

Before proceeding further, we give some lemmas to achieve the main results.

Lemma 1. *[36] For $\varphi(t) \in \mathbb{R}^p$, $\mathscr{P} = \mathscr{P}^T \in \mathbb{R}^{p\times p}$, $\mathscr{H} \in \mathbb{R}^{q\times p}$ and $rank(\mathscr{H}) = r < p$, two equivalent statements are shown as follows:*

a. $\varphi^T(t)\mathscr{P}\varphi(t) < 0$, *for any* $\varphi(t) \neq 0$, $\mathscr{H}\varphi(t) = 0$;
b. $\exists \mathscr{M} \in \mathbb{R}^{p\times q}$ *satisfying* $\mathrm{He}(\mathscr{M}\mathscr{H}) + \mathscr{P} < 0$.

Under the normal ETM (2), the corresponding results of ensuring the closed-loop system (10) to be asymptotically stable is given in Lemma 2.

Lemma 2. *For given positive scalars σ, α, ρ, the controller gain matrix K, the asymptotic stability of the system (10) is ensured under the normal ETM (2), if there exist symmetric matrices $P > 0$, $\Phi > 0$ and matrix G, such that*

$$\Theta = \begin{bmatrix} -He(G) & \Theta_{12} & -GB(K+\Delta_q) \\ & \Theta_{22} & -\alpha GB(K+\Delta_q) \\ & * & -\Phi \end{bmatrix} < 0, \tag{11}$$

where

$$\Theta_{12} = G(A + B(K+\Delta_q)) + P - \alpha G^T,$$
$$\Theta_{22} = \alpha He(G(A + B(K+\Delta_q))) + \sigma\Phi.$$

Proof. Let us select the Lyapunov functional as

$$V(t) = x^T(t)Px(t). \tag{12}$$

In order to ensure the system (10) be asymptotically stable, it is required that

$$\dot{V}(t) = 2x^T(t)P\dot{x}(t) < 0, \;\; t \in \Omega. \tag{13}$$

Combing the triggering condition (2) with (13), we have

$$\dot{V}(t) < 2x^T(t)P\dot{x}(t) + \sigma x^T(t)\Phi x(t) - e(t)^T\Phi e(t) < 0. \tag{14}$$

Then, the condition (14) can be rewritten as

$$\zeta^T(t)\Psi\zeta(t) < 0, \tag{15}$$

where

$$\zeta(t) = \begin{bmatrix} \dot{x}(t) \\ x(t) \\ e(t) \end{bmatrix}, \quad \Psi = \begin{bmatrix} 0 & P & 0 \\ & \sigma\Phi & 0 \\ & * & -\Phi \end{bmatrix}.$$

To be consistent with Lemma 1, we rewrite the closed-loop system (10) as

$$\mathscr{H}\zeta(t) = 0, \tag{16}$$

where $\mathscr{H} = \begin{bmatrix} -I & A + B(K+\Delta_q) & -B(K+\Delta_q) \end{bmatrix}$.
Thus, by employing Lemma 1 to (15), it yields

$$\zeta^T(t)\hat{\Psi}\zeta(t) = \zeta^T(t)(\Psi + \mathrm{He}(\mathscr{M}\mathscr{H}))\zeta(t) < 0, \tag{17}$$

where

$$\hat{\Psi} = \begin{bmatrix} -He(G) & \hat{\Psi}_{12} & \hat{\Psi}_{13} \\ & \hat{\Psi}_{22} & \hat{\Psi}_{23} \\ & * & -\Phi \end{bmatrix}, \quad \mathscr{M} = \begin{bmatrix} G \\ \alpha G \\ 0 \end{bmatrix},$$

$$\hat{\Psi}_{12} = G(A + B(K+\Delta_q)) + P - \alpha G^T,$$
$$\hat{\Psi}_{22} = \alpha He(G(A + B(K+\Delta_q))) + \sigma\Phi,$$
$$\hat{\Psi}_{13} = -GB(K+\Delta_q), \;\; \hat{\Psi}_{23} = -\alpha GB(K+\Delta_q),$$

which is equivalent to (19). □

Lemma 3. *[37] For given matrices $E_1 = E_1^T$, E_2 and E_3 with appropriate dimensions, one has $E_1 + E_2\Delta(t)E_3 + E_3^T\Delta^T(t)E_2^T < 0$ for all $\Delta(t)$ satisfying $\Delta^T(t)\Delta(t) \le I$, if and only if there exists a positive scalar ϖ such that*

$$E_1 + \varpi^{-1}E_2E_2^T + \varpi E_3^TE_3 < 0. \tag{18}$$

3. Main results

3.1. Stability analysis and syntheis under DETM

The stability analysis for the system (10) with the quantizer (6) and the DETM (3) is formulated in the following theorem.

Theorem 1. *For given positive scalars σ, α, ρ and the controller gain matrix K, the asymptotic stability of the system (10) is ensured under the DETM (3), if there exist symmetric matrices $P > 0$, $\Phi > 0$, and matrix G such that*

$$\Xi = \begin{bmatrix} -He(G) & \Xi_{12} & -GB(K+\Delta_q) \\ & \Xi_{22} & -\alpha GB(K+\Delta_q) \\ & * & -\Phi \end{bmatrix} < 0, \tag{19}$$

where

$$\Xi_{12} = G(A + B(K+\Delta_q)) + P - \alpha G^T,$$
$$\Xi_{22} = \alpha \mathrm{He}(G(A + B(K+\Delta_q))) + \sigma\Phi.$$

Proof. By left and right-multiplying (19) with $\zeta^T(t)$ and its transpose, it results in

$$\zeta^T(t)\Xi\zeta(t) < 0. \tag{20}$$

According to Lemma 1, one can obtain that (20) is equivalent to

$$\zeta^T(t)(\Xi - \mathrm{He}(\mathscr{M}\mathscr{H}))\zeta(t) < 0, \tag{21}$$

where

$$\mathscr{H} = \begin{bmatrix} -I & A + B(K+\Delta_q) & -B(K+\Delta_q) \end{bmatrix},$$

$$\mathscr{M}^T = \begin{bmatrix} G^T & \alpha GF^T & 0 \end{bmatrix}^T.$$

Then, the condition (21) is further expressed as

$$2x^T(t)P\dot{x}(t) + \sigma x^T(t)\Phi x(t) - e^T(t)\Phi e(t) < 0. \tag{22}$$

Based on the DETM (3), one can obtain

$$2\theta x^T(t)P\dot{x}(t) + e^T(t)\Phi e(t) - \sigma x^T(t)\Phi x(t) < 0. \tag{23}$$

Substituting (23) into (22), one has

$$2(1+\theta)x^T(t)P\dot{x}(t) < 0. \tag{24}$$

Select the Lyapunov functional as

$$V(t) = (1+\theta)x^T(t)Px(t). \tag{25}$$

Then, the asymptotic stability of the system (10) can be ensured if (26) is satisfied

$$\dot{V}(t) = 2(1+\theta)x^T(t)P\dot{x}(t) < 0,\ t \in \Omega. \tag{26}$$

It is seen that (26) is equivalent to (24), which completes the proof. □

Remark 4. Note that the stability conditions in Lemma 2 and Theorem 1 are equivalent, which means that the proposed DETM (3) does not introduce any extra conservativeness in the stability analysis comparing with the normal ETM (2). In contrast to $g(t) < 0$ in (2), however, $g(t)$ is not always required to be negative and relaxed to be less than a positive scalar $-\dot{V}(t)$ by (3).

The following proposition gives that, for a prescribed system state, the next triggering instant generated by a DETM is larger than that generated by a normal ETM.

Proposition 1. *Let $t_k = t_k^n = t_k^d$, t_{k+1}^n be the next triggering time generated by the normal ETM (2) and t_{k+1}^d be next triggering time generated by the DETM (3), then $t_{k+1}^n \le t_{k+1}^d$.*

Proof. Assuming that $t_{k+1}^n > t_{k+1}^d$ and substituting t_{k+1}^d into (2), we have

$$g(t_{k+1}^d) = f_2(t_{k+1}^d) < 0. \tag{27}$$

In terms of Lemma 2, one can derive

$$\dot{V}(t) + \sigma x^T(t)\Phi x(t) - e^T(t)\Phi e(t) < 0, \tag{28}$$

which leads to

$$f_1(t^d_{k+1}) - f_2(t^d_{k+1}) < 0. \tag{29}$$

By combing (27) and (29), it is obtained that

$$f_1(t^d_{k+1}) < f_2(t^d_{k+1}) < 0. \tag{30}$$

Based on the DETM (3), one can know that

$$f_2(t^d_{k+1}) > -\theta f_1(t^d_{k+1}) \geq 0, \tag{31}$$

which contradicts to (27). Hence, $t^n_{k+1} \leq t^d_{k+1}$. □

Based on Theorem 1, the design of the quantized event-triggered controller for the system (10) is developed in Theorem 2.

Theorem 2. *For given positive scalars σ, α, ρ, ϖ, the asymptotic stability of the system (10) is guaranteed under the DETM (3), if there exist symmetric matrices $\hat{P} > 0$, $\hat{\Phi} > 0$ and matrices N, X, such that*

$$\hat{\Xi} = \begin{bmatrix} -He(X) & \hat{\Xi}_{12} & \hat{\Xi}_{13} & \varpi z^2 B & 0 \\ & \hat{\Xi}_{22} & \hat{\Xi}_{23} & \alpha\varpi z^2 B & X \\ & * & -\hat{\Phi}_1 & 0 & -X \\ & * & * & -\varpi z^2 I & 0 \\ & * & * & * & -\varpi I \end{bmatrix} < 0, \tag{32}$$

where

$$\hat{P} = XPX^T, \ \ \hat{\Phi} = X\Phi X^T,$$
$$\hat{\Xi}_{12} = BN + AX^T + \hat{P} - \alpha X,$$
$$\hat{\Xi}_{22} = \alpha \mathrm{He}(AX^T + BN) + \sigma\hat{\Phi}_2,$$
$$\hat{\Xi}_{13} = -BN, \ \ \hat{\Xi}_{23} = -\alpha BN.$$

Moreover, one can obtain the controller gain as $K = NX^{-T}$.

Proof. The condition (19) can be written as

$$\Xi = \Lambda + \mathrm{He}\left(H^T_\Delta \Delta_q E_\Delta\right) < 0, \tag{33}$$

where

$$\Lambda = \begin{bmatrix} -He(G) & \Lambda_{12} & -GBK \\ & \Lambda_{22} & -\alpha GBK \\ & * & -\Phi \end{bmatrix},$$

$$\Lambda_{12} = G(A+BK) + P - \alpha G^T,$$
$$\Lambda_{22} = \alpha\mathrm{He}(G(A+BK)) + \sigma\Phi,$$

$$E_\Delta = \begin{bmatrix} 0 & I & -I \end{bmatrix},$$

$$H_\Delta = \begin{bmatrix} (GB)^T & \alpha(GB)^T & 0 \end{bmatrix}.$$

By using Lemma 3, it follows from (33) that there exist a scalar ϖ such that

$$\Lambda + \varpi H^T_\Delta \Delta^2_q H_\Delta + \varpi^{-1} E^T_\Delta E_\Delta < 0, \tag{34}$$

which equals to

$$\Lambda + \varpi z^2 H^T_\Delta H_\Delta + \varpi^{-1} E^T_\Delta E_\Delta < 0. \tag{35}$$

With the help of Schur complement, the condition (35) is equivalent to

$$\begin{bmatrix} \Lambda & \varpi z^2 H^T_\Delta & E^T_\Delta \\ & -\varpi z^2 I & 0 \\ & * & -\varpi I \end{bmatrix} < 0. \tag{36}$$

By pre and post-multiplying (36) with *diag*{X, X, X, I, I} and its transpose and setting $G = X^{-1}$, $\hat{P} = XPX^T$, $\hat{\Phi} = X\Phi X^T$, it results in (32).

Then, the proof is completed. □

Remark 5. By introducing a free variable G and a scalar α via Finsler lemma, the Lyapunov variable P is separated away from the controller gain K. Then, the controller gain K is obtained as $K = NX^{-T}$ instead of $K = NP^{-1}$, which removes the coupling between K and P and has the potential to reduce the controller design conservativeness.

B. Event-Triggered Control Systems with transmission delay

Considering the time-varying network-induced transmission delay τ_k with upper bound τ, one has $l_k = t_k + \tau_k$, $l_{k+1} = t_{k+1} + \tau_{k+1}$. The packet disorder is not considered in this paper, which means $l_{k+1} > l_k$. To utilize the DETM, an artificial delay $\tau(t) \in [0, \tau_M]$ satisfying (3) is chosen, which implies

$$\theta f_1(t) + \hat{f}_2(t) < 0, \ t \in [l_k, \ l_{k+1}) \tag{37}$$

where

$$f_1(t) = 2x^T(t)P\dot{x}(t),$$
$$\hat{f}_2(t) = \epsilon^T(t)\Phi\epsilon(t) - \sigma x^T(t-\tau(t))\Phi x(t-\tau(t)),$$
$$\epsilon(t) = x(t-\tau(t)) - x(t_k),$$
$$\tau(t) = \frac{\tau_{k+1} - t}{\tau_{k+1} - \tau_k}\tau_k + \frac{t - \tau_k}{\tau_{k+1} - \tau_k}\tau_{k+1}.$$

By combining the above triggering condition (37), the resulting closed-loop system with quantization and transmission delay is modeled as:

$$\dot{x}(t) = Ax(t) + B(K + \Delta_q)(x(t-\tau(t)) - \epsilon(t)). \tag{38}$$

For the system (38), the asymptotic stability condition is derived in Theorem 3. According to Theorem 3, the system synthesis is given in Theorem 4.

Theorem 3. *For given positive scalars σ, α, ρ, τ_M and the controller gain matrix K, the asymptotic stability of the system (38) is guaranteed under the DETM (3), if there exist symmetric matrices $P > 0$, $\Phi > 0$, $S > 0$, $R > 0$, and matrix F such that*

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & 0 & \Sigma_{15} \\ & \Sigma_{22} & \Sigma_{23} & 0 & \Sigma_{25} \\ & * & \Sigma_{33} & S & 0 \\ & * & * & -S & 0 \\ & * & * & * & -\Phi \end{bmatrix} < 0, \tag{39}$$

where

$$\Sigma_{11} = -\mathrm{He}(F) + \tau_M^2 S,$$
$$\Sigma_{12} = FA + P - \alpha F^T,$$
$$\Sigma_{13} = FB(K + \Delta_q),$$
$$\Sigma_{15} = -FB(K + \Delta_q),$$
$$\Sigma_{22} = \alpha \mathrm{He}(FA) + R - S,$$
$$\Sigma_{23} = \alpha FB(K + \Delta_q) + S,$$
$$\Sigma_{25} = -\alpha FB(K + \Delta_q),$$
$$\Sigma_{33} = \sigma\Phi - R - 2S.$$

Proof. Pre- and post-multiplying (39) with $\hat{\zeta}^T(t)$ and $\hat{\zeta}(t)$, respectively, one can get

$$\hat{\zeta}^T(t)\Sigma\hat{\zeta}(t) < 0, \tag{40}$$

where

$$\hat{\zeta}(t) = \begin{bmatrix} \dot{x}^T(t) & x^T(t) & x^T(t-\tau(t)) & x^T(t-\tau_M) & \epsilon^T(t) \end{bmatrix}^T.$$

Applying Lemma 1 to (40), one has

$$\hat{\zeta}^T(t)\big(\Sigma - \mathrm{He}(\hat{\mathscr{M}}\hat{\mathscr{H}})\big)\hat{\zeta}(t) < 0, \tag{41}$$

which can be rewritten as

$$\begin{aligned} & 2x^T(t)P\dot{x}(t) + x^T(t)Rx(t) - x^T(t-\tau(t))Rx(t-\tau(t)) \\ & + \tau_M^2\dot{x}^T(t)S\dot{x}(t) - \begin{bmatrix} x(t) \\ x(t-\tau(t)) \\ x(t-\tau_M) \end{bmatrix}^T \mathbf{S} \begin{bmatrix} x(t) \\ x(t-\tau(t)) \\ x(t-\tau_M) \end{bmatrix} \\ & + \sigma x^T(t-\tau(t))\Phi x(t-\tau(t)) - \epsilon^T(t)\Phi\epsilon(t) < 0, \end{aligned} \tag{42}$$

where

$$\hat{\mathscr{H}} = \begin{bmatrix} -I & A & B(K+\Delta_q) & 0 & -B(K+\Delta_q) \end{bmatrix},$$

$$\hat{\mathscr{M}} = \begin{bmatrix} F^T & \alpha F^T & 0 & 0 & 0 \end{bmatrix}^T,$$
$$\mathbf{S} = \begin{bmatrix} S & -S & 0 \\ & 2S & -S \\ & * & S \end{bmatrix}.$$

By utilizing the technical lemma in [37] derived based on the well-known Jensen inequality, (42) can be ensured by

$$\begin{aligned} &2x^T(t)P\dot{x}(t) + x^T(t)Rx(t) + \tau^2\dot{x}^T(t)S\dot{x}(t) \\ &- x^T(t-\tau(t))Rx(t-\tau(t)) - \tau_M\int_{t-\tau_M}^{t}\dot{x}^T(s)S\dot{x}(s)ds \\ &+ \sigma x^T(t-\tau(t))\Phi x(t-\tau(t)) - \epsilon^T(t)\Phi\epsilon(t) < 0, \end{aligned} \tag{43}$$

Considering the triggering condition (37), one derives that (43) is guaranteed once

$$\begin{aligned} &(1+\theta)2x^T(t)P\dot{x}(t) + x^T(t)Rx(t) - x^T(t-\tau_M)Rx(t-\tau_M) \\ &+ \tau_M^2\dot{x}^T(t)S\dot{x}(t) - \tau_M\int_{t-\tau_M}^{t}\dot{x}^T(s)S\dot{x}(s)ds < 0. \end{aligned} \tag{44}$$

By integrating the left hand side of (44), it yields

$$\begin{aligned} \mathcal{L} \triangleq\ &(1+\theta)x^T(t)Px(t) + \int_{t-\tau_M}^{t} x^T(s)Rx(s)ds \\ &+ \int_{-\tau_M}^{0}\int_{t+\beta}^{t}\dot{x}^T(s)S\dot{x}(s)dsd\beta, \end{aligned} \tag{45}$$

which is positive definite based on $\theta > 0$, $P > 0$, $R > 0$ and $S > 0$.

According to the well-known Lyapunov stability theory, system (38) is asymptotically stable if there exists a Lyapunov functional $V(t)$ satisfying $V(t) > 0$ and $\dot{V}(t) < 0$.

Thus, we chose $\mathcal{L}$ as the Lyapunov functional:

$$\begin{aligned} V(t) = \mathcal{L} = &(1+\theta)x^T(t)Px(t) + \int_{t-\tau_M}^{t} x^T(s)Rx(s)ds \\ &+ \int_{-\tau_M}^{0}\int_{t+\beta}^{t}\dot{x}^T(s)S\dot{x}(s)dsd\beta, \end{aligned} \tag{46}$$

then $V(t) > 0$ and $\dot{V}(t) < 0$ are ensured by (44) and (45), respectively. This renders the system (38) asymptotically stable. □

Remark 6. To deal with the integral term $\tau_M\int_{t-\tau_M}^{t}\dot{x}^T(s)S\dot{x}(s)ds$ induced by the Lyapunov functional, a conventional method based Jensen inequality is utilized in this paper, which could lead to some conservatism. Recently, some improved inequalities such as Wirtinger-based inequality, Bessel-Legendre inequality and their combination with reciprocally convex lemma have been proposed to reduce such conservatism, which will be studied in our future work.

Theorem 4. *For given positive scalars σ, α, ρ, τ_M and ϖ, the asymptotic stability of the system (38) is ensured under the DETM (3), if there exist symmetric matrices $\hat{P} > 0$, $\hat{\Phi} > 0$, $\hat{S} > 0$, $\hat{R} > 0$, and matrices N, Y such that*

$$\hat{\Sigma} = \begin{bmatrix} \hat{\Sigma}_{11} & \hat{\Sigma}_{12} & \hat{\Sigma}_{13} & 0 & \hat{\Sigma}_{15} & \varpi z^2B & 0 \\ & \hat{\Sigma}_{22} & \hat{\Sigma}_{23} & 0 & \hat{\Sigma}_{25} & \alpha\varpi z^2B & 0 \\ & * & \hat{\Sigma}_{33} & \hat{S} & 0 & 0 & Y \\ & * & * & -\hat{S} & 0 & 0 & 0 \\ & * & * & * & -\hat{\Phi} & 0 & -Y \\ & * & * & * & * & -\varpi z^2I & 0 \\ & * & * & * & * & * & -\varpi I \end{bmatrix} < 0, \tag{47}$$

where

$$\begin{aligned} \hat{\Sigma}_{11} &= -\mathrm{He}(Y) + \tau_M^2\hat{S}, \\ \hat{\Sigma}_{12} &= AY^T + \hat{P} - \alpha Y, \\ \hat{\Sigma}_{13} &= BN,\ \ \hat{\Sigma}_{15} = -BN, \\ \hat{\Sigma}_{22} &= \alpha\mathrm{He}(AY^T) + \hat{R} - \hat{S}, \\ \hat{\Sigma}_{23} &= \alpha BN + \hat{S},\ \ \hat{\Sigma}_{25} = -\alpha BN, \\ \hat{\Sigma}_{33} &= \sigma\hat{\Phi} - \hat{R} - 2\hat{S}, \end{aligned}$$

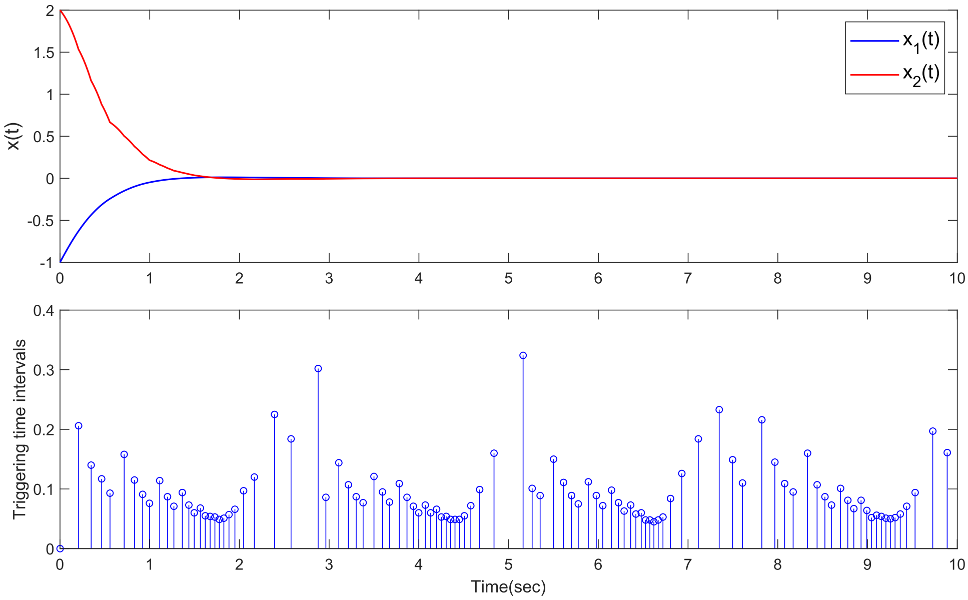

Fig. 2. The state $x(t)$ and triggering time intervals under the normal ETM (2).

Table 1
The average triggering time interval h.

ETMs	h
Normal ETM (2)	0.0980
DETM (3) ($\theta = 0.1$)	0.1220
DETM (3) ($\theta = 1$)	0.2778
DETM (3) ($\theta = 10$)	0.3846
DETM (3) ($\theta = 100$)	0.4348

$$\hat{P} = YPY^T, \ \ \hat{\Phi} = Y\Phi Y^T,$$
$$\hat{S} = YSY^T, \ \ \hat{R} = YRY^T.$$

Then, one can derive the controller gain as $K = NY^{-T}$.

Proof. By taking the same process with the proof of Theorem 2, (47) is derived easily. □

4. Experimental example

Example 1. Consider a linear networked system (1) with state matrices

$$A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}, \ \ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

By taking $\sigma = 0.1, \ \alpha = 1, \ \varpi = 1, \ \rho = 0.8$ and solving Theorem 2, the corresponding parameters are obtained as

$$P = \begin{bmatrix} 6.8466 & 1.3754 \\ 1.3754 & 2.1503 \end{bmatrix}, \Phi = \begin{bmatrix} 18.4306 & 8.7015 \\ 8.7015 & 23.1939 \end{bmatrix},$$
$$K = \begin{bmatrix} -2.5853 & -6.3777 \end{bmatrix}.$$

With the initial condition $x(0) = \begin{bmatrix} -1 & 2 \end{bmatrix}^T$, the curves of the state and the average triggering time intervals under the proposed DETM (3) with different $\theta = 1$ and normal ETM (2) are drawn in Figs. 2 and 3. The average triggering time intervals for different θ are given in Table 1.

From these figures, one observes that the asymptotic stability of the system (10) is satisfied with the designed event-triggered controllers under the normal ETM (2) with 92 events and the DETM (3) with 43 events. This shows that the proposed DETM can reduce the triggering events dramatically compared with the normal ETM. Furthermore, according to Table 1, it is noted that the larger θ is chosen, the larger average triggering time interval is obtained by the DETM, which shows that θ plays a similar role with the triggering threshold σ in the normal ETM.

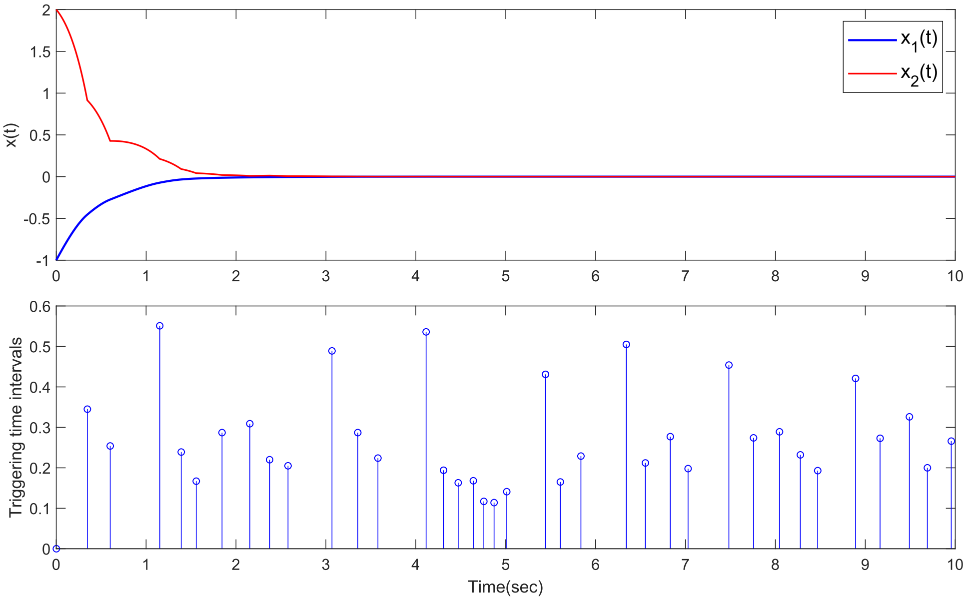

Fig. 3. The state $x(t)$ and triggering time intervals under the DETM (3) with $\theta = 1$.

Table 2
The average triggering time interval h.

ETMs	h
Normal ETM (2)	0.1613
DETM (3) ($\theta = 0.1$)	0.2083
DETM (3) ($\theta = 1$)	0.3448
DETM (3) ($\theta = 100$)	0.4545

Example 2. We consider the network-induced time-varying transmission delay with upper bound $\tau_M = 0.05$ and an inverted pendulum on a cart borrowed from Yue et al. [23] with following parameters:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\dfrac{Mg}{m} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \dfrac{g}{l} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \dfrac{1}{m} \\ 0 \\ -\dfrac{1}{ml} \end{bmatrix},$$

with $m = 1$, $M = 10$, $l = 3$ and $g = 9.8$.

For $\sigma = 0.1$, $\alpha = 1$, $\varpi = 1000$ and $\rho = 0.9$, the corresponding parameters are obtained as

$$P = \begin{bmatrix} 0.0115 & 0.0163 & 0.1219 & 0.0708 \\ 0.0163 & 0.0332 & 0.2582 & 0.1500 \\ 0.1219 & 0.2582 & 2.3150 & 1.3357 \\ 0.0708 & 0.1500 & 1.3357 & 0.7732 \end{bmatrix},$$

$$\Phi = \begin{bmatrix} 0.0387 & 0.0752 & 0.8534 & 0.4903 \\ 0.0752 & 0.1807 & 2.0245 & 1.1630 \\ 0.8534 & 2.0245 & 24.1247 & 13.8348 \\ 0.4903 & 1.1630 & 13.8348 & 7.9418 \end{bmatrix},$$

$$K = \begin{bmatrix} 18.7010 & 46.8448 & 589.5152 & 337.2417 \end{bmatrix}.$$

Under the initial condition $x(0) = \begin{bmatrix} 0.98 & 0 & 2 & 0 \end{bmatrix}^T$ and $\theta = 0.1,\ 1,\ 100$, the corresponding figures of the DETM (3) and the normal ETM (2) are depicted in Figs. 4–7. The average triggering time intervals are given in Table 2.

According to Figs. 4–7, it is demonstrated that the designed controller can render the system with time-varying transmission delay to be stable for the normal ETM (2) and the DETM (3), respectively. It is also observed that under the similar convergence time, the proposed DETM with $\theta = 0.1$, $\theta = 1$ and $\theta = 100$ can reduce 22.58%, 53.22% and 66.67% of the trigger-

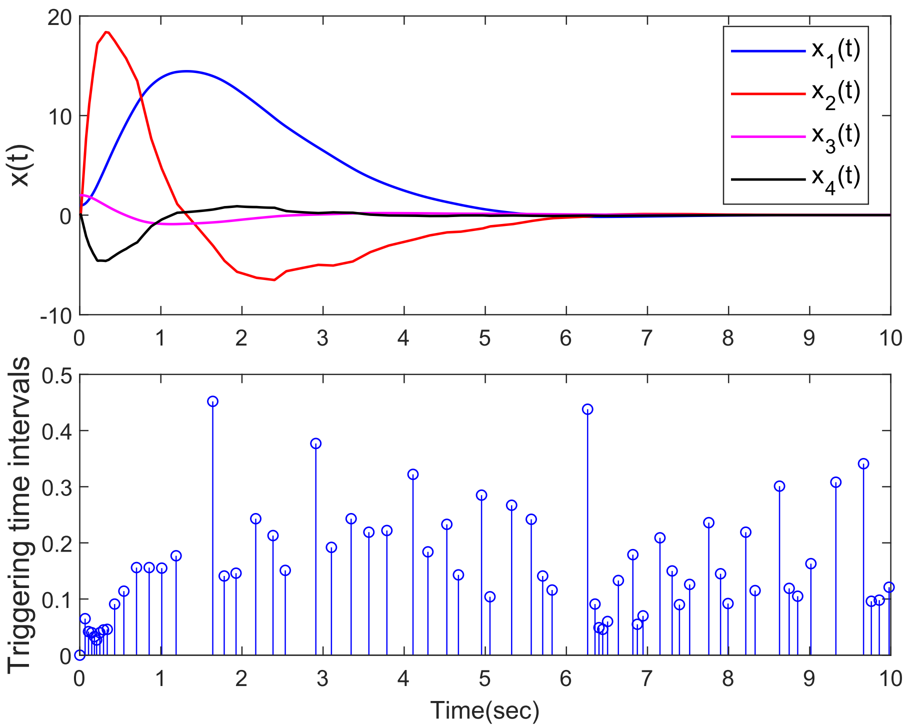

Fig. 4. The state $x(t)$ and triggering time intervals under the normal ETM (2) with $\tau_M = 0.05$.

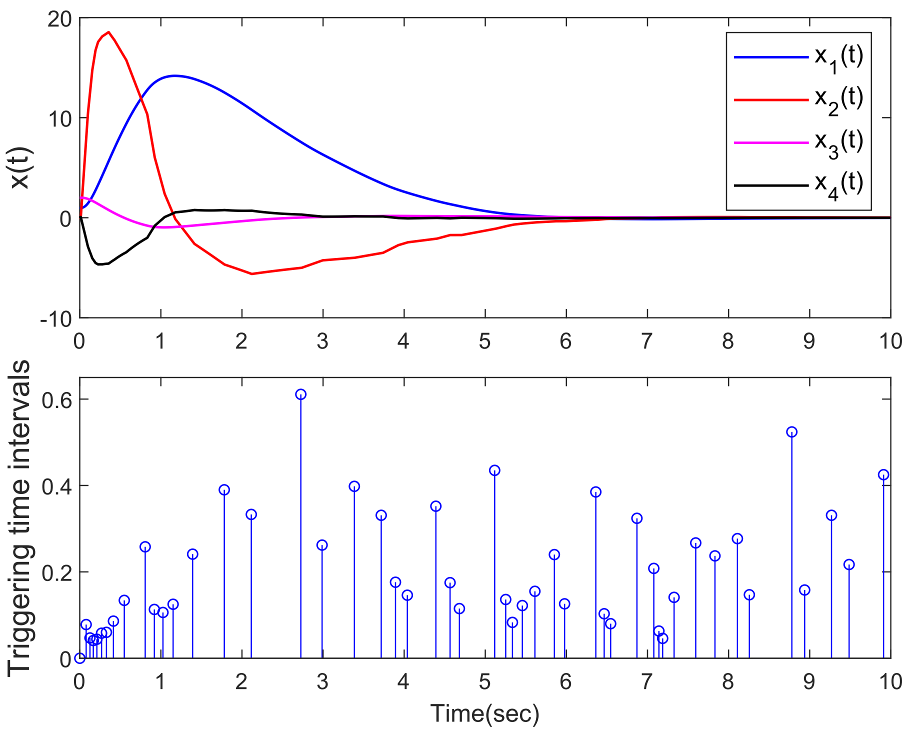

Fig. 5. The state $x(t)$ and triggering time intervals under the DETM (3) with $\theta = 0.1$ and $\tau_M = 0.05$.

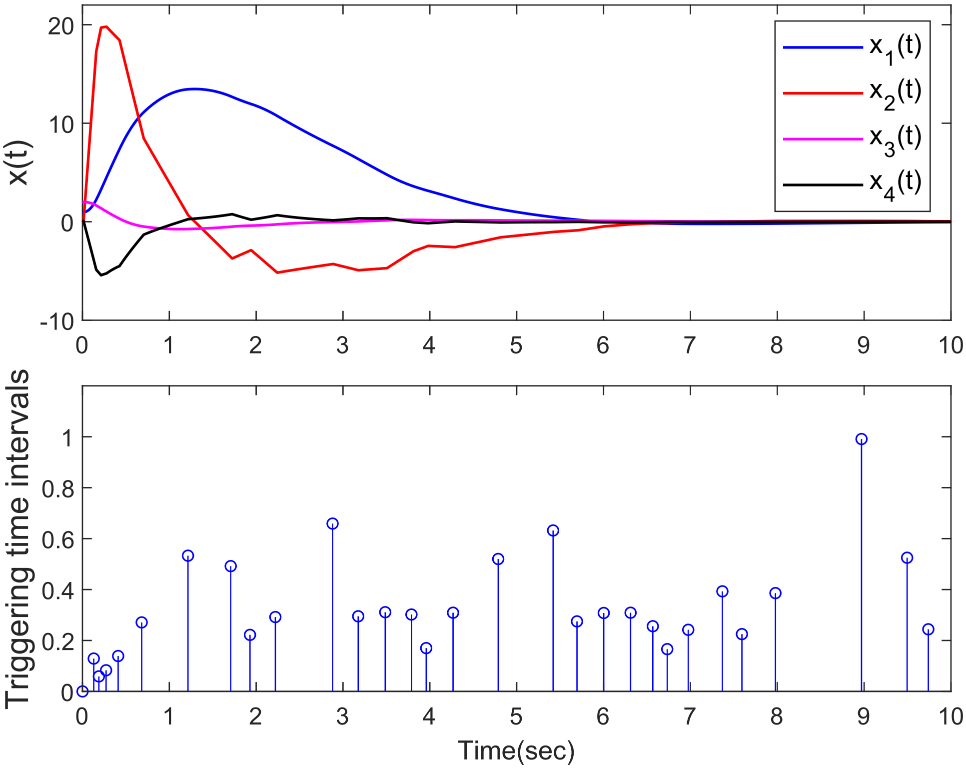

Fig. 6. The state $x(t)$ and triggering time intervals under the DETM (3) with $\theta = 1$ and $\tau_M = 0.05$.

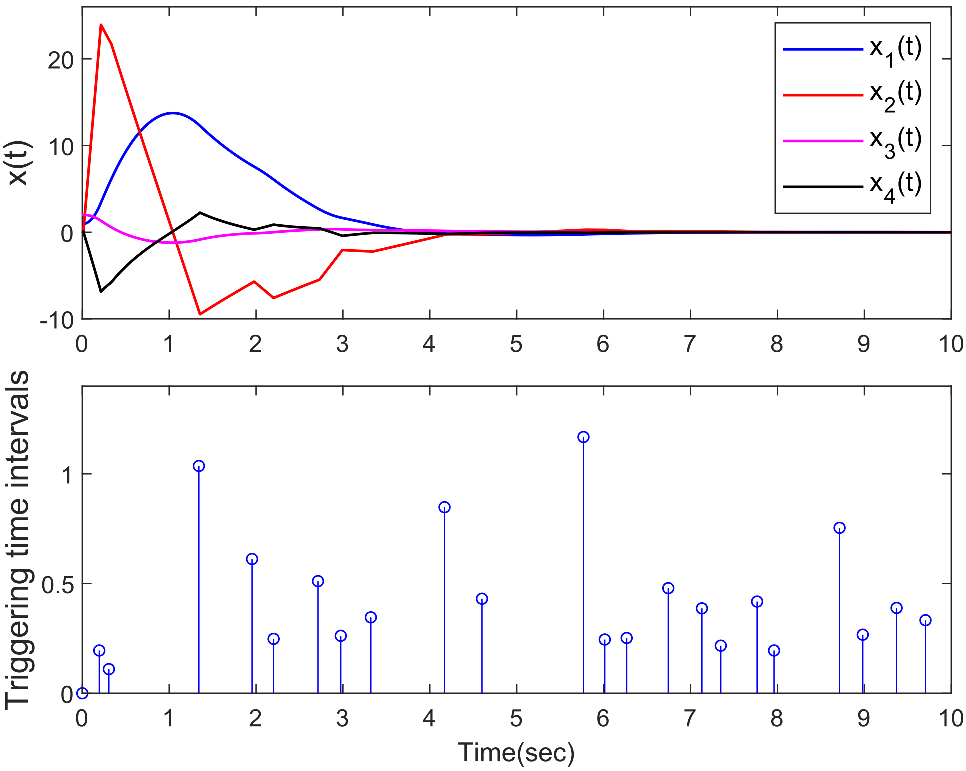

Fig. 7. The state $x(t)$ and triggering time intervals under the DETM (3) with $\theta = 100$ and $\tau_M = 0.05$.

ing events compared to the normal ETM, respectively. Moreover, Table 2 also gives the conclusion that the average triggering time interval h is increasing along with the increase of θ in our DETM (3). These indicate the advantage of the proposed DETM for saving network resources than normal ETM.

5. Conclusion

This study contributes to the derivative-based event-triggered control for linear networked systems with quantization and transmission delay. Compared with the normal ETM, the proposed DETM introduces an extra derivative term, which can lead to less conservative results and larger inter-event times. Sufficient LMI conditions are established to the existence of a stabilization quantized event-triggered controller for the closed-loop system. Two numerical examples are simulated to show the effectiveness of the presented approach. In the future, the proposed DETM will be extended to some complicate systems, such as stochastic systems [38], nonlinear systems [39] and singularly perturbed systems [40].

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